University of Bern
Institute of Computer Science
Cryptology and Data Security

Vertex Coloring in the Hybrid Network Model

BSc, MSc Thesis

Project Description

Distributed computing often revolves around efficiently solving problems whose input is distributed among nodes of a network with as little communication as possible among the nodes. A famous graph problem is the so called vertex coloring problem. Here, we are given a graph $G = (V, E)$ and a (sufficiently large) set of colors $\{1, \dots, k\}$. The problem is to compute a *proper coloring*, i.e., assign a color in $\{1, \dots, k\}$ to each vertex $v \in V$ such that any two adjacent nodes in G have different colors. This graph problem can be seen as a distributed problem as follows. The nodes of the network correspond to the vertices of the graph G and each node initially only knows its neighbors in G, that is, the edge set E is not known by a central instance, but rather distributed among nodes. The problem is solved as soon as each node outputs its color in a proper coloring of G. The goal is to minimize the amount of communication among nodes to solve this problem; and for this we have to define a proper model how this communication works.

In this project we would like to consider the HYBRID model of distributed computing. Here, we assume computationally unbounded nodes that have unique IDs and communicate in synchronous *rounds* (which is our basic unit of time). In each round, each node can exchange messages with other nodes using two fundamentally different *modes of communication* (hence the name HYBRID). First, nodes can use a *local communication mode* where they may exchange arbitrarily large messages but *only* with their neighbors in a communication graph G (this communication graph also doubles as the problem input). Second, nodes can use a *global communication mode* where they can exchange a message with any other node, but only a small amount of information may be exchanged this way in each round ($O(\log^2 n)$ bits per node). The goal is that nodes exchange messages to find their proper color with as few rounds as possible. The problem of coloring G with the *minimal* number of colors is relatively hard (i.e., takes many rounds) using *only either* the local *or* the global communication mode. The hope (and goal of this project) is that by combining both modes of communication one can get a solution that is strictly faster than if one of communication modes is used in isolation.

Recommended Reading

- Introduction of Linial, Locality in distributed graph algorithms, Journal on Computing, 1992
- Introduction of Augustine at al., Shortest paths in a hybrid network model, SODA, 2020

Requirements

- Basic lectures on algorithms and math
- Curiosity about the topic

Contact

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